

Further Pure Mathematics FP3 Mark scheme

Question	Scheme		Marks
1	$y = 9 \cosh x + 3 \sinh x + 7x$		
	$\frac{dy}{dx} = 9 \sinh x + 3 \cosh x + 7$	Correct derivative	B1
	$9 \frac{(e^x - e^{-x})}{2} + 3 \frac{(e^x + e^{-x})}{2} + 7 = 0$	Replaces $\sinh x$ and $\cosh x$ by the correct exponential forms	M1
	Note that the first 2 marks can score the other way round: M1: $y = 9 \frac{(e^x + e^{-x})}{2} + 3 \frac{(e^x - e^{-x})}{2} + 7x$ B1: $\frac{dy}{dx} = 9 \frac{(e^x - e^{-x})}{2} + 3 \frac{(e^x + e^{-x})}{2} + 7$		
	$12e^{2x} + 14e^x - 6 = 0$ oe	M1: Obtains a quadratic in e^x A1: Correct quadratic	M1 A1
	$(3e^x - 1)(2e^x + 3) = 0 \Rightarrow e^x = \dots$	Solves their quadratic as far as $e^x = \dots$	M1
	$x = \ln\left(\frac{1}{3}\right)$	cso (Allow $-\ln 3$) $e^x = -\frac{3}{2}$ need not be seen. Extra answers, award A0	A1
	Alternative		
	$\frac{dy}{dx} = 9 \sinh x + 3 \cosh x + 7$	Correct derivative	B1
	$9 \sinh x = -3 \cosh x - 7 \Rightarrow 81 \sinh^2 x = 9 \cosh^2 x + 42 \cosh x + 49$		
	$72 \cosh^2 x - 42 \cosh x - 130 = 0$	Squares and attempts quadratic in $\cosh x$	M1
	$(3 \cosh x - 5)(12 \cosh x + 13) = 0 \Rightarrow \cosh x = \frac{5}{3}$	M1: Solves quadratic A1: Correct value	M1 A1
	$x = \ln\left(\frac{5}{3} \pm \sqrt{\left(\frac{5}{3}\right)^2 - 1}\right)$	Use of $\ln$ form of arcosh	M1
	$x = \ln\left(\frac{1}{3}\right)$	cso (Allow $-\ln 3$)	A1
	NB: Ignore any attempts to find the y coordinate		
(6 marks)			

Question	Scheme		Marks
2(a)	$\frac{x^2}{25} + \frac{y^2}{4} = 1, \quad P(5 \cos \theta, 2 \sin \theta)$		
	$\frac{dx}{d\theta} = -5 \sin \theta, \quad \frac{dy}{d\theta} = 2 \cos \theta$ or $\frac{2x}{25} + \frac{2y}{4} \frac{dy}{dx} = 0$	Correct derivatives or correct implicit differentiation	B1
	$\frac{dy}{dx} = \frac{2 \cos \theta}{-5 \sin \theta}$	Divides their derivatives correctly or substitutes and rearranges	M1
	$M_N = \frac{5 \sin \theta}{2 \cos \theta}$	Correct perpendicular gradient rule	M1
	$y - 2 \sin \theta = \frac{5 \sin \theta}{2 \cos \theta} (x - 5 \cos \theta)$	Correct straight line method (any complete method) Must use their gradient of the normal.	M1
	$5x \sin \theta - 2y \cos \theta = 21 \sin \theta \cos \theta^*$	cso	A1*
			(5)
(b)	At Q, $x = 0 \Rightarrow y = -\frac{21}{2} \sin \theta$		B1
	M is $\left(\frac{0 + 5 \cos \theta}{2}, \frac{2 \sin \theta - \frac{21}{2} \sin \theta}{2} \right)$ $\left(= \left(\frac{5}{2} \cos \theta, -\frac{17}{4} \sin \theta \right) \right)$	Correct mid-point method for at least one coordinate Can be implied by a correct x coordinate	M1
	L cuts x-axis at $\frac{21}{5} \cos \theta$		B1
	Area $OPM = OLP$ $+OLM$ $\frac{1}{2} \cdot \frac{21}{5} \cos \theta \cdot 2 \sin \theta + \frac{1}{2} \cdot \frac{21}{5} \cos \theta \cdot \frac{17}{4} \sin \theta$	M1: Correct triangle area method using their coordinates A1: Correct expression	M1 A1
	$= \frac{105}{16} \sin 2\theta$	Or $6.5625 \sin 2\theta$ must be positive	A1
			(6)

Question	Scheme		Marks
2(b) <i>continued</i>	Alternative 1: Using Area OPM		
	See above for B1M1		B1 M1
	Area $\Delta OPM = \frac{1}{2} \begin{vmatrix} 0 & 5 \cos \theta & \frac{5}{2} \cos \theta & 0 \\ 0 & 2 \sin \theta & -\frac{17}{4} \sin \theta & 0 \end{vmatrix}$	M1: Correct determinant with their coords, with 2 or 3 points. $\begin{smallmatrix} 0 \\ 0 \end{smallmatrix}$ should be at both or neither end. A1: Correct determinant (There are more complicated determinants using the 3 points.)	M1 A1
	$= \frac{1}{2} \left(0 + 5 \sin \theta \cos \theta + 0 - 0 + \frac{85}{4} \sin \theta \cos \theta - 0 \right)$	A1	A1
	$= \frac{105}{4} \sin \theta \cos \theta$		
	$= \frac{105}{16} \sin 2\theta$		A1
			(6)
	Alternative 2: Using Area OPQ		
	At Q , $x = 0 \Rightarrow y = -\frac{21}{2} \sin \theta$		B1
	Area $\Delta OPQ = \frac{1}{2} \begin{vmatrix} 5 \cos \theta & 0 \\ 2 \sin \theta & -\frac{21}{2} \sin \theta \end{vmatrix}$	Can be implied by the following line	M1 A1
	$= \frac{1}{2} \times \frac{105}{2} \sin \theta \cos \theta$	OQ is base, x coord of P is height	A1
	$= \frac{105}{8} \sin 2\theta$		
	Area $OPM = \frac{1}{2}$ Area OPQ		M1
	$= \frac{105}{16} \sin 2\theta$		A1
			(6)

Question	Scheme	Marks
2(b) <i>continued</i>	Alternative 3	
	At Q , $x = 0 \Rightarrow y = -\frac{21}{2} \sin \theta$	B1
	M is $\left(\frac{0+5\cos\theta}{2}, \frac{2\sin\theta - \frac{21}{2}\sin\theta}{2} \right) \quad \left(= \left(\frac{5}{2}\cos\theta, -\frac{17}{4}\sin\theta \right) \right)$	M1
	$OP = \sqrt{4\sin^2\theta + 25\cos^2\theta} \quad (= \sqrt{4+21\cos^2\theta})$	B1
	$d = \frac{\frac{5}{2}\cos\theta \times \frac{2\sin\theta}{5\cos\theta} + \frac{17}{4}\sin\theta}{\sqrt{\frac{4\sin^2\theta}{25\cos^2\theta} + 1}} = \frac{\frac{21}{4}\sin\theta}{\sqrt{\frac{4+21\cos^2\theta}{25\cos^2\theta}}}$	
	$\text{Area} = \frac{1}{2} \times \frac{\frac{21}{4}\sin\theta}{\sqrt{\frac{4+21\cos^2\theta}{25\cos^2\theta}}} \times \sqrt{4+21\cos^2\theta}$	M1 A1
	$= \frac{105}{16} \sin 2\theta$	A1
		(6)
(11 marks)		

Question	Scheme		Marks
3(a)	$x^2 + 4x + 13 \equiv (x + 2)^2 + 9$		B1
	$\int \frac{1}{(x+2)^2 + 9} dx = \frac{1}{3} \arctan\left(\frac{x+2}{3}\right)$	M1: $\arctan f(x)$.	M1 A1
		A1: Correct expression	
	$\left[\frac{1}{3} \arctan\left(\frac{x+2}{3}\right) \right]_{-2}^1 = \frac{1}{3} (\arctan 1 - \arctan 0)$	Correct use of limits $\arctan 0$ need not be shown	M1
	$\frac{\pi}{12}$	cao	A1
			(5)
	Alternative		
	Sub $x + 2 = 3 \tan t$		
	$x^2 + 4x + 13 \equiv (x + 2)^2 + 9$		B1
	$\frac{dx}{dt} = 3 \sec^2 t \quad x = -2, \tan t = 0, t = 0; x = 1, \tan t = 1, t = \frac{\pi}{4}$		
	$\int \frac{3 \sec^2 t}{9 \tan^2 t + 9} dt = \frac{1}{3} \int dt = \frac{1}{3} t$	M1 sub and integrate inc use of $\tan^2 + 1 = \sec^2$ A1 Correct expression Ignore limits	M1 A1
	$\left[\frac{\pi}{12} \right]_0^{\frac{\pi}{4}}$	Either change limits and substitute Or reverse substitution and substitute original limits	M1
	$\frac{\pi}{12}$	cao	A1
			(5)

Question	Scheme		Marks
3(b)	$4x^2 - 12x + 34 = 4\left(x - \frac{3}{2}\right)^2 + 25$ or $(2x - 3)^2 + 25$	M1: $4(x \pm p)^2 \pm q, (p, q \neq 0)$ A1: $4\left(x - \frac{3}{2}\right)^2 + 25$	M1 A1
	$\int \frac{1}{\sqrt{4\left(x - \frac{3}{2}\right)^2 + 25}} \mathrm{d}x = \frac{1}{2} \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 + \frac{25}{4}}} \mathrm{d}x = \frac{1}{2} \operatorname{arsinh}\left(\frac{x - \frac{3}{2}}{\frac{5}{2}}\right)$ M1: $k \operatorname{arsinh} f(x)$. A1: Correct expression		M1 A1
	$\left[\frac{1}{2} \operatorname{arsinh}\left(\frac{x - \frac{3}{2}}{\frac{5}{2}}\right)\right]_{-1}^4 = \frac{1}{2}(\operatorname{ar sinh}(1) - \operatorname{ar sinh}(-1))$	Correct use of limits	M1
	$= \frac{1}{2}(\ln(1 + \sqrt{2}) - \ln(-1 + \sqrt{2}))$	Uses the logarithmic form of arsinh	M1
	$= \frac{1}{2} \ln(3 + 2\sqrt{2})$ or $\ln(1 + \sqrt{2})$	cao	A1
			(7)
	Alternative: Second M1 A1		
	Sub $2x - 3 = u$ or $2x - 3 = 5 \sinh u$		
	$\int_{\operatorname{arsinh}(-1)}^{\operatorname{arsinh}1} \frac{1}{\sqrt{25 \sinh^2 u + 25}} 5 \cosh u \mathrm{d}u = \left[\frac{1}{2} \operatorname{arsinh}\left(\frac{u}{5}\right)\right]_{-5}^5$	M1 A1	
	$\int_{-5}^5 \frac{1}{2\sqrt{u^2 + 25}} \mathrm{d}u = \left[\frac{1}{2} \operatorname{arsinh}\left(\frac{u}{5}\right)\right]_{-5}^5$		
(12 marks)			

Question	Scheme		Marks
4(a)	$\mathbf{M} = \begin{pmatrix} 1 & k & 0 \\ -1 & 1 & 1 \\ 1 & k & 3 \end{pmatrix}$		
	$ \mathbf{M} = 3 - k - k(-3 - 1)(= 3k + 3)$	Correct determinant in any form	B1
	$\mathbf{M}^T = \begin{pmatrix} 1 & -1 & 1 \\ k & 1 & k \\ 0 & 1 & 3 \end{pmatrix} \text{ or minors } \begin{pmatrix} 3 - k & -4 & -k - 1 \\ 3k & 3 & 0 \\ k & 1 & 1 + k \end{pmatrix}$ $\text{or cofactors } \begin{pmatrix} 3 - k & 4 & -k - 1 \\ -3k & 3 & 0 \\ k & -1 & 1 + k \end{pmatrix}$		B1
	$\mathbf{M}^{-1} = \frac{1}{3 + 3k} \begin{pmatrix} 3 - k & -3k & k \\ 4 & 3 & -1 \\ -k - 1 & 0 & 1 + k \end{pmatrix}$	M1: Identifiable full attempt at inverse including reciprocal of determinant . Could be indicated by at least 6 correct elements.	M1 A1ft A1ft
		A1ft: Two rows or two columns correct (follow through their determinant but not incorrect entries in the matrices used)	
		A1ft: Fully correct inverse (follow through as before)	
NB: If every element is the negative of the correct element, allow M1A1A0			
			(5)
(b)	$\mathbf{MN} = \begin{pmatrix} 3 & 5 & 6 \\ 4 & -1 & 1 \\ 3 & 2 & -3 \end{pmatrix} \Rightarrow \mathbf{N} = \mathbf{M}^{-1} \begin{pmatrix} 3 & 5 & 6 \\ 4 & -1 & 1 \\ 3 & 2 & -3 \end{pmatrix}$	Correct statement	B1
	$\mathbf{N} = \frac{1}{3} \begin{pmatrix} 3 & 0 & 0 \\ 4 & 3 & -1 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & 5 & 6 \\ 4 & -1 & 1 \\ 3 & 2 & -3 \end{pmatrix} = \begin{pmatrix} 3 & 5 & 6 \\ 7 & 5 & 10 \\ 0 & -1 & -3 \end{pmatrix}$	M1: Multiplies the given matrix by their $\mathbf{M}^{-1}$ in the correct order Must include the " $\frac{1}{3}$ "	M1 A(2, 1, 0)
		A2: Correct matrix (−1 each error). If left with $\frac{1}{3}$ outside the matrix award A0	
			(4)
(9 marks)			

Question	Scheme		Marks
5(a)	$y = \operatorname{artanh}(\cos x)$		
	$\frac{dy}{dx} = \frac{1}{1 - \cos^2 x} \times -\sin x$	Correct use of the chain rule	M1
	$= \frac{-\sin x}{\sin^2 x} = \frac{-1}{\sin x} = -\operatorname{cosec} x$ *	A1: Correct completion with no errors	A1
			(2)
	Alternative 1		
	$\tanh y = \cos x \Rightarrow \operatorname{sech}^2 y \frac{dy}{dx} = -\sin x$		
	$\frac{dy}{dx} = \frac{-\sin x}{\operatorname{sech}^2 y} = \frac{-\sin x}{1 - \cos^2 x}$	Correct differentiation to obtain a function of x	M1
	$= \frac{-\sin x}{\sin^2 x} = \frac{-1}{\sin x} = -\operatorname{cosec} x$ *	A1: Correct completion with no errors	A1
			(2)
	Alternative 2		
	$\operatorname{artanh}(\cos x) = \frac{1}{2} \ln \left(\frac{1 + \cos x}{1 - \cos x} \right)$		
	$\frac{dy}{dx} = \frac{1}{2} \times \frac{1 - \cos x}{1 + \cos x} \times \frac{-\sin x(1 - \cos x) - \sin x(1 + \cos x)}{(1 - \cos x)^2}$	Correct differentiation to obtain a function of x	M1
	$= \frac{-2 \sin x}{2(1 - \cos^2 x)} = -\operatorname{cosec} x$ *	A1: Correct completion with no errors	A1
			(2)
(b)	$\int \cos x \operatorname{artanh}(\cos x) dx = \sin x \operatorname{artanh}(\cos x) - \int \sin x \times -\operatorname{cosec} x dx$ M1: Parts in the correct direction A1: Correct expression		M1 A1
	$\left[\sin x \operatorname{artanh}(\cos x) + x \right]_0^{\frac{\pi}{6}} = \frac{1}{2} \operatorname{artanh} \left(\frac{\sqrt{3}}{2} \right) + \frac{\pi}{6} (- (0))$ M1: Correct use of limits on either part (provided both parts are integrated). Lower limit need not be shown		M1
	$= \frac{1}{4} \ln \left(\frac{1 + \frac{\sqrt{3}}{2}}{1 - \frac{\sqrt{3}}{2}} \right) + \frac{\pi}{6}$	Use of the logarithmic form of artanh	M1
	$= \frac{1}{4} \ln(7 + 4\sqrt{3}) + \frac{\pi}{6}$ or $\frac{1}{2} \ln(2 + \sqrt{3}) + \frac{\pi}{6}$	Cao (oe)	A1
	The last 2 M marks may be gained in reverse order.		(5)
	(7 marks)		

Question	Scheme		Marks
6(a)	$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$	Two correct vectors in Π Can be negatives of those shown	B1
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & 1 & 1 \\ 1 & -1 & 3 \end{vmatrix} = \begin{pmatrix} 4 \\ 7 \\ 1 \end{pmatrix}$	M1: Attempt cross product of two vectors lying in Π (At least one no. to be correct.)	M1 A1
		A1: Correct normal vector	
	$\begin{pmatrix} 4 \\ 7 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 4 + 14 + 3$	Attempt scalar product with their normal and a point in the plane	dM1
	$4x + 7y + z = 21$	Cao (oe)	A1
			(5)
	Alternative 1		
	$a + 2b + 3c = d$ $-a + 3b + 4c = d$ $2a + b + 6c = d$	Correct equations	B1
	$a = \frac{4}{21}d, b = \frac{1}{3}d, c = \frac{1}{21}d$	M1: Solve for a, b and c in terms of d	M1 A1
		A1: Correct equations	
	$d = 21 \Rightarrow a = \dots, b = \dots, c = \dots$	Obtains values for a, b, c and d	M1
	$4x + 7y + z = 21$	Cao (oe)	A1
			(5)
	Alternative 2: Using $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$ where $\mathbf{b}$ and $\mathbf{c}$ are vectors in Π		
	Two correct vectors in the plane	See main scheme	B1
	Eg $\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + s \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$		M1
	$x = 1 - 2s + t$ $y = 2 + s - t$ $z = 3 + s + 3t$	Deduce 3 correct equations	A1
	$4x + 7y + z = 21$	M1: Eliminate s, t A1: Cao	M1 A1
			(5)

Question	Scheme		Marks
6(b)	$AD \cdot AB \times AC$	Attempt suitable triple product	M1
	$= \begin{pmatrix} 4 \\ 7 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} k-1 \\ 2 \\ 11 \end{pmatrix} = 4k - 4 + 14 + 11$		
	$\therefore \frac{1}{6}(4k + 21) = 6$	M1: Set $\frac{1}{6}$ (their triple product) = 6	dM1 A1
		A1: Correct equation	
	$k = \frac{15}{4}$	Cao (oe)	A1
			(4)
	Alternative		
	Area ABC $= \frac{1}{2} \overrightarrow{AB} \overrightarrow{AC} = \frac{1}{2} \sqrt{6} \sqrt{11}$	Attempt area ABC and distance between D and II	M1
	D to II is $\frac{4k + 28 + 14 - 21}{\sqrt{16 + 49 + 1}}$		
	$\frac{1}{6} \sqrt{6} \sqrt{11} \frac{4k + 28 + 14 - 21}{\sqrt{16 + 49 + 1}} = 6$	M1: Set $\frac{1}{3}$ (their area x their distance) = 6	dM1 A1
		A1: Correct equation	
	$k = \frac{15}{4}$	Cao (oe)	A1
			(4)
			(9 marks)

Question	Scheme		Marks
7(a)	$x = 3t^4, \quad y = 4t^3$		
	$\frac{dx}{dt} = 12t^3, \quad \frac{dy}{dt} = 12t^2$	Correct derivatives	B1
	$S = (2\pi) \int y \left(\left(\frac{dx}{dt} \right)^2 + \left(\frac{dy}{dt} \right)^2 \right)^{\frac{1}{2}} dt = (2\pi) \int 4t^3 \sqrt{(12t^3)^2 + (12t^2)^2} dt$ $\left(= (2\pi) \int 4t^3 (144t^6 + 144t^4)^{\frac{1}{2}} dt \right)$		M1
	M1: Substitutes their derivatives into a correct formula (2π not required)		
	$S = (2\pi) \int 4t^3 (144t^4)^{\frac{1}{2}} (t^2 + 1)^{\frac{1}{2}} dt$	Attempt to factor out at least t^4 - numerical factor may be left	M1
	$S = 96\pi \int_0^1 t^5 (t^2 + 1)^{\frac{1}{2}} dt$	Correct completion	A1
			(4)
(b)	$u^2 = t^2 + 1 \Rightarrow 2u \frac{du}{dt} = 2t \text{ or } 2u = 2t \frac{dt}{du}$	Correct differentiation	B1
	$t = 0 \Rightarrow u = 1, \quad t = 1 \Rightarrow u = \sqrt{2}$	Correct limits Alternative: Reverse the substitution later. (Treat as M1 in this case and award later when work seen)	B1
	$S = (96\pi) \int t^5 \times u \times \frac{u}{t} du$		
	$S = (96\pi) \int (u^2 - 1)^2 \times u^2 du$	M1: Complete substitution	M1 A1
		A1: Correct integral in terms of u . Ignore limits, need not be simplified	
	$S = (96\pi) \int (u^6 - 2u^4 + u^2) du = (96\pi) \left[\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} \right]$		dM1
	M1: Expands and attempts to integrate		
	$S = 96\pi \left[\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} \right]_1^{\sqrt{2}} = 96\pi \left\{ \left(\frac{\sqrt{2}^7}{7} - \frac{2\sqrt{2}^5}{5} + \frac{\sqrt{2}^3}{3} \right) - \left(\frac{1}{7} - \frac{2}{5} + \frac{1}{3} \right) \right\}$		ddM1
	M1: Correct use of their changed limits (both to be changed) Alternative: If sub reversed, substitute the original limits		
	$S = \frac{192\pi}{105} (11\sqrt{2} - 4)$	Cao eg $\frac{64\pi}{35}$	A1
			(7)
			(11 marks)

Question	Scheme		Marks
8(a)	$I_n = \int_0^{\ln 2} \tanh^{2n} x \, dx, \quad n \geq 0$		
	$\tanh^{2n} x = \tanh^{2(n-1)} x \tanh^2 x$		B1
	$\tanh^{2n} x = \pm \tanh^{2(n-1)} x (1 - \operatorname{sech}^2 x)$		M1
	$I_n = \int_0^{\ln 2} \tanh^{2(n-1)} x \, dx - \int_0^{\ln 2} \tanh^{2(n-1)} x \operatorname{sech}^2 x \, dx$		
	$I_n = I_{n-1} - \left[\frac{1}{2n-1} \tanh^{2n-1} x \right]_0^{\ln 2}$	M1: Correctly substitutes for I_{n-1} and obtains $\int \tanh^{2(n-1)} x \operatorname{sech}^2 x \, dx = k \tanh^{2n-1} x$	M1 A1
		A1: Correct expression	
	$= I_{n-1} - \frac{1}{2n-1} \left(\frac{3}{5} \right)^{2n-1} *$	Correct completion with no errors	A1*
			(5)
	Alternative		
	$I_n - I_{n-1} = \int_0^{\ln 2} (\tanh^{2n} x - \tanh^{2(n-1)} x) \, dx$		
	$= \int_0^{\ln 2} \tanh^{2(n-1)} x (\tanh^2 x - 1) \, dx$		B1
	$= \int_0^{\ln 2} \tanh^{2(n-1)} x (-\operatorname{sech}^2 x) \, dx$	$= \int_0^{\ln 2} \tanh^{2(n-1)} x (\pm \operatorname{sech}^2 x) \, dx$	M1
	$I_n - I_{n-1} = - \left[\frac{1}{2n-1} \tanh^{2n-1} x \right]_0^{\ln 2}$	M1: Obtains $\int \tanh^{2(n-1)} x \operatorname{sech}^2 x \, dx = k \tanh^{2n-1} x$	M1 A1
		A1: Correct expression	
	$= I_{n-1} - \frac{1}{2n-1} \left(\frac{3}{5} \right)^{2n-1} *$	Correct completion with no errors	A1*
			(5)

Question	Scheme		Marks
8(b)	$I_0 = \ln 2$	The integration must be seen.	B1
	$I_2 = I_1 - \frac{1}{3}\left(\frac{3}{5}\right)^3$	Applies the reduction formula once	M1
	$I_2 = I_0 - \frac{1}{1}\left(\frac{3}{5}\right)^1 - \frac{1}{3}\left(\frac{3}{5}\right)^3$	M1: Second application of the reduction formula	M1A1
		A1: Correct expression	
	$I_2 = \ln 2 - \frac{84}{125}$	cao	A1
	Special Case: If I_4 is found award B1 for I_0 or I_1 and M1M0A0A0		
			(5)
	Alternative		
	$I_1 = \int_0^{\ln 2} \tanh^2 x \, \mathrm{d}x = \int_0^{\ln 2} (1 - \operatorname{sech}^2 x) \mathrm{d}x$		
	$I_1 = \left[x - \tanh x \right]_0^{\ln 2}$	Correct integration	B1
	$I_2 = I_1 - \frac{1}{3}\left(\frac{3}{5}\right)^3$	Applies the reduction formula once	M1
	$I_1 = \ln 2 - \tanh(\ln 2) = \ln 2 - \frac{3}{5}$	M1: Uses limits	M1A1
		A1: Correct expression	
	$I_2 = \ln 2 - \frac{3}{5} - \frac{1}{3}\left(\frac{3}{5}\right)^3$		
	$= \ln 2 - \frac{84}{125}$		A1
			(5)
(10 marks)			

